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學 術 論 文

Monopolistic Pricing

with Consumers′ Information Exchange

about "Product Match" on the Internet

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National Center for Economic Research
at Tsinghua University

No.200012　　2000 年 3 月

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Abstract

In this paper, we analyze the optimal pricing strategies of a monopoly firm selling digital products such as computational software on the internet when consumers learn from each other about product match. We show that the consumers′ learning process moves the firm′s optimal price closer to the real product benefits if the consumers are homogeneous. If the consumers are heterogeneous, we find that depending on the consumers' prior beliefs, the firm has the incentive of building a larger or smaller consumer base to promote or reduce the effects of consumers' word-of-mouth communication about product match.

KEY WORDS: monopoly, word-or-mouth, pricing

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消费者可以在 INTERNET 上交流产品信息情况下的垄断定价分析

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论文摘要

在这篇文章中我们分析了在 INTERNET 上销售电子产品（如电脑软件等）的垄断企业的最优价格策略。我们假设消费者在购买和使用产品前不知道它是否能满足他们的偏好，但他们可以向购买过该产品的消费者学习他们使用这个产品的经验。我们发现：如果消费者具有相同的偏好，他们之间的信息交流会使厂商的最优价格接近产品的实际效益。如果消费者的偏好不同，厂商会通过最优价格策略来建立或大或小的消费者群体，以便利用消费者间的信息交流而获取更大的利润。

Monopolistic Pricing with Consumers′ Information Exchange about "Product Match" on the Internet[1]

Li Mingzhi and Xu Hai

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1 INTRODUCTION

The rapid expansion of the internet has brought to consumers huge amounts of products information. On the one hand, firms may set up homepages to demonstrate product features, provide information such as the amount of sales up to the minute and consumers' reactions. On the other hand, information exchange among consumers can also become an important source for them to learn product benefits. Consumers may post their opinions voluntarily onto an online bulletin board or in a chat room due to goodwill or just simply to express their happiness or anger. The success story of Amazon.com is an good example of firms' taking advantage of word of mouth communication among consumers on the electronic commerce markets. As commented by Jeff Bezos, Amazon.com′s founder and CEO, "The very best way to promote oneself online is by word of mouth. This is because one person can tell 5000 people something as easily as he can tell five people. This is what has happened with Amazon.com." [2]

In this paper, we analyze the optimal intertemporal pricing strategies of a monopoly firm selling digital products such as computational software on the internet. Since these products are experience goods, the consumers are uncertain about how well the products will match their needs. The goods are also durable, therefore repeat purchases play no role in the process of the consumers learning about the product quality or suitability. An important channel through which the consumers may gain additional information about the products is word-of-mouth communication.

Learning from others about product quality or product match has been investigated by Beggs (1994), McFadden and Train (1996). Beggs assumes that a consumer will know the product quality perfectly if he can find one consumer who has tried the product; otherwise his beliefs about the product quality remain unchanged. We assume that the consumers can observe the aggregate result of a group of experienced consumers. This is a more reasonable assumption considering the unique characteristics of information exchange among consumers on the internet. It is hard to believe that a consumer will completely believe one particular stranger's opinion posted on the internet. However, the aggregation of other consumers' opinions

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[2] "Amazon.com′s Amazing Allure", *Publishers Weekly*, November 4, 1996, Page 24--25

does send a signal about the product quality and may change the consumer′s beliefs (e.g. when the consumer knows "the law of large numbers"). McFadden and Train examine the influences of others' experiences of a product on a consumer's beliefs about product benefits, and the consumer's tradeoff between trying the product himself and waiting to observe others' experiences. However, we concentrate on the optimal strategies of the firm, which lends quite a different flavor to the analysis.

When consumers face new products offered on the internet, they may have some ideas about the product benefits from the firm′s advertisements such as an on-line multimedia demonstration of the product features. However, the consumers still have the incentives of collecting more information about the product; especially for an experience good[3], the quality of which can not be verified before purchase (especially when a refund or return is not possible).

We are interested in the firm′s optimal pricing strategies when the consumers learn from each other about product benefits. Multiple time periods are needed to capture the essential nature of the phenomena. In the first period, the consumers make purchase decisions based on their prior beliefs about the product benefits. In the second period, new consumers enter the market and observe the experiences of the first period consumers. These ′new′ consumers make their purchase decisions based on their updated beliefs using Bayes' theorem. Within each period, consumers may have heterogeneous tastes for the product. Since the product is durable, the consumers do not make repeat purchases. Therefore the consumers in both periods are all "new customers" who have no prior experiences with the product.

Our model is based on the one used by Tirole (1988) to analyze the firms' optimal intertemporal prices with consumers′ repeat purchases. Uncertainty about the product is assumed to be a dichotomous match. If a good match occurs the consumer will enjoy some positive benefits from the product and if a bad match occurs the benefits to the consumer are zero. We believe this characterization captures the salient features of a broad range of quality uncertainty problems on the EC marketplaces. An accurate online financial report about the stock market may lead a consumer to make the right move of selling or buying stocks and an inaccurate forecast may lead to a huge financial disaster. A right choice of computational software will satisfy a buyer's future needs perfectly and a wrong choice may simply waste money and turn out to be useless. While Tirole assumes the probability of a product match between the product and the consumers′ preferences to be common knowledge, we introduce uncertainties into this simple model and investigate the impacts of consumers' learning from each other about product match on the firm′ optimal pricing strategies.

The paper is organized as follows. In section 2, assuming that the consumers are homogeneous, we derive the monopolist′s optimal intertemporal pricing decisions. We show that if the consumers are optimistic about the product benefits, consumers′ learning from others′ experiences leads the firm to charge a higher price in the first period and lower the price in the second period. If the consumers are pessimistic, the firm would do the reverse. In either case, the consumers′ learning process pushes the

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[3] The characteristics of experience goods are discussed in Tirole (1988), page 106.

firm′ optimal price to the true product benefits. In section 3, we introduce heterogeneity among consumers' tastes and into the simple model in section 2. Assuming that there are continuous types of consumers in each time period, we show that depending on the consumers' prior beliefs, the firm has the incentive of building a larger or smaller consumer base to promote or reduce the effects of consumers' word-of-mouth communication about product match. Some concluding remarks close the paper in section 4.

2 PRICING WITH HOMOGENEOUS CONSUMERS

In this section we consider the monopoly firm′s optimal intertemporal pricing strategies facing homogeneous consumers who have the same real probability of matches between their tastes and the product as well as the same beliefs on the product matches. They will also enjoy the same benefits from consuming the product if product matches occur. The ability that ′new′ consumers can observe the experiences of ′old′ consumers will change the firm′s pricing strategies. To see this, let us begin from the characterization of the consumers′ uncertainty about the product match in a single-period model.

2.1 A One-Period Model

As in Tirole (1988, page 116), we set up the following matching model. A consumer with taste θ ($\theta>0$) has the following preferences for the good produced by the monopolist:

$$u = \begin{cases} \theta s - p & \text{if he buys at p} \\ 0 & \text{otherwise} \end{cases}, \qquad (1)$$

where s denotes the quality to the consumer. There are two potential levels of quality: s = 0 ("no match") and s = 1 ("match"). The probability of a match between the consumer and the product is x^*, belonging to (0,1). Unlike Tirole (1988), we assume that the consumers do not know the true value of x^*; however, each consumer has a prior belief on x^*. From Bayesian concepts, this belief could be represented as a generalized probability density function $f(x)$. Since we assume homogeneity among the consumers, the taste variable θ is the same for all consumers.

For the following analyses we assume the representative consumer′s prior on x^* (denoted as x^B) is a beta distribution with parameters α and β[4], then

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[4] There are two primary reasons that we choose a beta distribution for the following analyses:

(1) The family of beta distribution is a conjugate family for samples from a Bernoulli distribution.

(2) When $\alpha = 1$ and $\beta = 1$, the beta distribution becomes the uniform distribution on the interval (0,1). These properties simplify the analyses tremendously.

$$+ \quad f(x \mid \alpha,\ \beta) = \begin{cases} \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)+\Gamma(\beta)} x^{\alpha}(1-x)^{\beta} & \text{for } x \in (0,1) \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where

$$\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy\,,\ \ \Gamma(\beta) = \int_0^\infty y^{\beta-1} e^{-y} dy \ \text{ and}$$

$$\Gamma(\alpha+\beta) = \int_0^\infty y^{\alpha+\beta-1} e^{-y} dy$$

Given this distribution, the mean value of the consumer′s belief on x^* is:

$$E(x^B|\alpha,\beta) = \frac{\alpha}{\alpha+\beta}$$

Without any additional information, the consumer′s expected value of the product benefits is:

$$\theta E(x^B|\alpha,\beta) = \theta \frac{\alpha}{\alpha+\beta},$$

and he will purchase the product if and only if $\theta \frac{\alpha}{\alpha+\beta} \geq p$.

For simplicity we also assume the cost of producing the product is 0. The firm will charge the optimal price according to the consumer′s expected benefit from the product: $p^* = \theta \frac{\alpha}{\alpha+\beta}$.

2.2 A Two-Period Model

Now suppose instead that the firm is selling the product in two time periods. We assume there are the same number of consumers in the two periods (denoted as N), and their tastes are identical as well. We also assume that all the consumers exist for only one period; therefore in either period no consumer has ever tried the product or known the product benefits. The first period consumers make their purchase decisions purely based on their prior beliefs. After purchasing and testing the product, these consumers know whether the product is a match or not, and voluntarily report the results, say, into an online newsgroup or onto an online bulletin board. The second period consumers update their beliefs on the product benefits based on these observations. Their purchase decisions are made according to the updated beliefs.

Without information exchange among the consumers, the firm′s optimal intertemporal pricing decision is straightforward. In each period, the firm charges a price according to the consumer′s expected benefits from consuming the product. Since we assume the consumers have the same characteristics in both time periods, the optimal prices in the two periods are simply duplications of the optimal one-period price discussed in section 2.1.

Now let us discuss the firm′s optimal intertemporal prices with consumers′ learning from others′ experiences. The characterizations of the consumers in the two periods are the same as defined in section 2.1, except that the second period consumers can observe the first period consumers′ posted results of product matches.

In the first period, the firm either charges

$$p_1 = \theta \frac{\alpha}{\alpha+\beta} \qquad (3)$$

and sells to all the consumers, or does not make a sale at all. By choosing no sale in the first period, the maximal price the firm can charge in the second period is

$$p_2 = \theta \frac{\alpha}{\alpha+\beta},$$

and the expected second period profit is

$$Np_2 = N\theta \frac{\alpha}{\alpha+\beta},$$

which is the same with the firm′s first period profit from charging

$$p_1 = \theta \frac{\alpha}{\alpha+\beta}$$

and selling to all the consumers. Obviously, if the firm has a time discount factor $\delta \le 1$, no sale in the first period cannot be more profitable than selling to all the consumers in the first period. We conclude that the firm′ optimal price in the first period is:

$$p_1 = \theta \frac{\alpha}{\alpha+\beta} \qquad (4)$$

For each consumer i of the N consumers who have tried the product in the first period, the observed result x_i is distributed according to the following probability density function:

$$x_i = \begin{cases} 1 & \text{with probability} \quad x^* \\ 0 & \text{with probability} \quad 1-x^* \end{cases} \qquad (5)$$

Assuming that each consumer′s product match is independent of any of the other consumers, the sequence of the N product matches can be regarded as a Bernoulli trial with parameter x^*, the results of which could be observed by a second period consumer.

Out of the N i.i.d. samples, the sum $N_1 = x_1 + x_2 + ... + x_N$ has a binomial distribution:

$$f(n|N,x^*) = \begin{cases} \binom{N}{n} x^{*N_i}(1-x^*)^{N-N_i} & \text{for} \quad n = 0, 1, 2, ..., N \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

A second period consumer′s updated belief can be calculated using the following results from [Degroot (1972), pp 160]. Suppose that X_1, ... X_n is a random sample from a Bernoulli distribution with an unknown value of the parameter W. Suppose also that the prior distribution of W is a beta distribution with parameter α and β such that α>0 and β>0. Then the posterior distribution of W when $X_i = x_i$ (i =1 ,..., n) is a beta distribution with parameters α+y and β+n-y, where $y = \sum_{i=1}^{n} x_i$.

A second period consumer′s posterior belief on x^* becomes a Beta distribution with parameters $\alpha+N_1$ and $\beta+(N-N_1)$. The consumer′s expected benefit from the product is

$$E\left(\frac{\alpha+N_1}{\alpha+N_1+\beta+(N-N_1)}\right) = \theta\frac{\alpha+Nx^*}{\alpha+\beta+N},$$

(because in average $E(N_1) = Nx^*$). Therefore the expected second period price the firm could charge is:

$$E(p_2) = E\left(\theta\frac{\alpha+N_1}{\alpha+N_1+\beta+(N-N_1)}\right) = \theta\frac{\alpha+Nx^*}{\alpha+\beta+N}.$$

(7)

Then we have

$$E(p_2)-p_1 = \theta\frac{\alpha+Nx^*}{(\alpha+\beta+N)} - \theta\frac{\alpha}{\alpha+\beta} = \theta N\frac{x^* - \frac{\alpha}{\alpha+\beta}}{(\alpha+\beta+N)}. \tag{8}$$

The result $E(p_2) > p_1$ holds if and only if $x^* > \frac{\alpha}{\alpha+\beta}$. This has proved the following proposition.

Proposition: If the consumers have optimistic prior beliefs on the product matches, $\frac{\alpha}{\alpha+\beta} > x^*$, the learning process makes the firm lower its second period price. If the learning consumers have pessimistic prior beliefs on the product matches, $\frac{\alpha}{\alpha+\beta} < x^*$, the learning process makes the firm raise its second period price. If

$\frac{\alpha}{\alpha+\beta} = x^*$, the consumers have unbiased prior beliefs on the probability of product matches, the learning process does not affect the firm′s pricing behavior, the firm′s expected second period price is the same as its first period price.

These results make obvious intuitive sense. The old consumers′ experiences of the product tell some truth about the product match. Since all the consumers are identical, a new consumer′s observation of N old consumers′ experiences of the product plays the same role as if he learns the probability of product match by making N independent samplings and observing the percentage of "s = 1" ("match"). The sampling results will correct the consumer′s beliefs toward the true parameter (x^*). As N goes to infinity, we could see that a second period consumer′s beliefs on the probability of product match go to the true probability, which is consistent with the law of large numbers.

Since the firm charges a price according to the consumers' beliefs on the expected value of the product, it is easy to infer the impacts of the consumers′ learning process on the firm′s total profits. If the consumers have optimistic prior beliefs on the product match, the learning process makes the firm′s profits lower than without consumers′ learning from one another. If the consumers have pessimistic prior beliefs on the product match, the learning process makes the firm′ profits higher than without consumers′ learning.

We implicitly assume that the firm knows the consumers' beliefs, but it would be interesting to know what prices the firm should charge if it does not know the consumers' beliefs. In essence, the approach of analysis will be similar to the one we are using here. In the first period, the firm will charge a price according to the firm′s beliefs of the consumer′s beliefs of the product′s expected benefits. And in the second period, the firm needs to figure out the consumer′s updated beliefs using Bayesian formula and accordingly charge the second period price.

3 Pricing with Heterogeneous Consumers

3.1 Model Set-Ups

The previous problem was made simple because the inference problem was very simple. The second period consumers collect data of the first period consumers' experiences of the product, and update their beliefs about the product match. In each period, the firm charges a price according to the consumers' beliefs about the product benefits. Since we assume homogeneous consumers, the updated beliefs are closer to the true probability of a product match.

3.2 Optimal Pricing with Continuous Types of Consumers

In this section, we consider the case of continuous types of consumers.

The model set-ups are the same with those in the previous sections except that the consumers' types are distributed according a continuous density function. For simplicity we assume that the consumers' types, θ s, are uniformly distributed on [0,1]. Each consumer of type θ has a prior belief on the real probability of product match, x^*, which is assumed to be a beta distribution, $B(\alpha(\theta),\beta(\theta))$ with mean,

$$m(\theta)=\frac{\alpha(\theta)}{\alpha(\theta)+\beta(\theta)}$$

and variance,

$$v(\theta)=\frac{\alpha(\theta)\beta(\theta)}{(\alpha(\theta)+\beta(\theta))^2(1+\alpha(\theta)+\beta(\theta))}$$

We also assume that the number of consumers in each time period to be N.

Let be θ_1 the such that $\theta_1 m(\theta_1)=p_1$, where $p_1$is the firm's first period price. The first period demand for the product is

$$N_1=N(1-\theta_1).$$

Assuming N_1^* out of the N_1 consumers are matched with the product, we know N_1^* has a binomial distribution with density function:

$$f(n|N_1,x^*)=\begin{cases}\begin{pmatrix}N_1^*\\ x\end{pmatrix}n^{x^*}(1-n)^{1-x^*} & \text{for } n=1,2,\ldots,N_1\\ 0 & \text{otherwise}\end{cases}$$

Then the mean value of N_1^* is N_1x^*.

According to the Bayesian Rule, in the second time period, consumer θ's mean value of product match will be updated to:

$$\frac{\alpha(\theta)+N_1x^*}{\alpha(\theta)+\beta(\theta)+N_1},$$

which is consumer θ's willingness to pay for the product.

Define θ_2 as the θ which satisfies:

$$\frac{\alpha(\theta_2)+N_1x^*}{\alpha(\theta_2)+\beta(\theta_2)+N_1}=\frac{p_2}{\theta_2},$$

where p_2 is the firm's second period price. Then the second period demand for the product will be $N(1-\theta_2)$.

Assuming that the time discount factor is equal to 1, the firm's total profit in the two periods are:

$$\pi(p_1, p_2) = p_1N(1-\theta_1) + p_2N(1-\theta_2)$$

We have transformed the firm's optimal pricing strategies to a dynamic programming problem **(**)** :

(**) $$\max_{p_1,p_2} \pi(p_1, p_2) = p_1N(1-\theta_1) + p_2N(1-\theta_2)$$

s.t 1. $$\theta_1\frac{\alpha(\theta_1)}{\alpha(\theta_1)+\beta(\theta_1)} = p_1$$

2. $$\frac{\alpha(\theta_2)+N(1-\theta_1)x^*}{\alpha(\theta_2)+\beta(\theta_2)+N(1-\theta_1)}=\frac{p_2}{\theta_2}$$

To solve this problem, we need a specification of $\alpha(\theta)$ and $\beta(\theta)$. We consider a simple case such that all the consumers have the same prior beliefs on the product match, $\alpha(\theta)=\beta(\theta)=1$; i.e. every consumer believes that the probability of product match is uniformly distributed on [0,1].

With this specification, problem ** is changed to:

$$\max_{p_1,p_2} \pi(p_1, p_2) = p_1N(1-\theta_1) + p_2N(1-\theta_2) \quad (9)$$

s.t. $$\frac{1}{2}\theta_1 = p_1 \quad (10)$$

$$\theta_2\frac{1+N(1-\theta_1)x^*}{2+N(1-\theta_1)} = p_2 \quad (11)$$

Pluging (10) and (11) into the objective function (9), from the first order conditions we get:

$$\theta_1 = \frac{1}{2} + \frac{1}{4}\frac{(1-2x^*)N}{[2+N(1-\theta_1)]^2}$$

$$\theta_2 = \frac{1}{2}$$

Then we have the following results:

if $x^* > 1/2$, $\theta_1 < 1/2$; if $x^* > 1/2$, $\theta_1 > 1/2$ and if $x^* = 1/2$, $\theta_1 = 1/2$.

From these we can see the effects of consumers' learning from each other. If the consumers are pessimistic about the product match in that their beliefs of the probability of product match are smaller than the true probability, the firm will build a smaller consumer base in the first period to reduce the effect of word of mouth communication among consumers. If the consumers are optimistic, the firm will do the reverse. If the consumers have unbiased beliefs, the learning process has no effect on the firm's pricing strategy.

As a comparison, Tirole (1988) shows that if the probability of product match is common knowledge, the firm will serve the same size of consumer base. The firm is better off resting on his laurels and milking his goodwill.

4 CONCLUSIONS

The analysis in this paper is an initial attempt to incorporate information exchange among consumers into the analyses of market equilibria. It is our beliefs that consumers are playing more and more active roles in reducing quality uncertainties in the electronic commerce marketplaces compared with in the traditional marketplaces. The advances of information technology make consumers information exchange at ever-declining costs and unprecedented speed. It is thus important to explore the impacts of this new phenomenon. We have shown that depending on the tastes of the consumers and their prior beliefs, word of mouth communication can become an important tool for the firm to extract more profits. The firm has the incentive of promoting or reducing the effects of word of mouth communication among the consumers. Some limitations in the models are prominent and left for future research.

First, we assume the information exchange among consumers is costless. In reality, the consumers need to make an investment to get connected to the internet, and they may also have to pay fees to enter a suitable information resource, such as an online chat-room. Besides, the experience of surfing the internet may not be so pleasant due to the slow traffic of information flow and a huge amount of junk information. It is worthwhile to investigate the impacts of the consumers' costs of search for valuable information on the market equilibria. A second limitation is that the model consists of two periods and assumes that the consumers have the same beliefs. Introducing more periods and heterogeneous beliefs among consumers will be an interesting extension.

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