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Ex-ante fairness under constrained school choice: an experimental approach

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Abstract

In a college admission mechanism, students often match with colleges by using a noisy signal of their true abilities (e.g. the total score in an exam). The matching outcome thus may be imperfect on ex-ante fairness, which suggests matching students with higher abilities to better colleges. Previous literature (e.g., Lien et al. (2016, 2017)) suggest preference submission before exam (i.e., before the signal is generated) to impove ex-ante fairness, but the effect is ambiguous. In this paper we consider constraining student choice over colleges to (further) improve ex-ante fairness. We design treatments with different constraint levels under Boston and Serial Dictatorship mechanisms, with preference submission before or after exam. Constraining student choice increases the probability of achieving ex-ante fairness under Boston and SD mechanism with preference submission before exam. However, under those mechanisms, the probability of achieving highly unfair matching is also increased, resulting in a more risky matching outcome.

Keywords: Constrained school choice Ex-ante fairness Matching Experiments

1. Introduction

In college admission mechanisms, colleges often observe only one or several noisy signals (e.g. exam scores) of students' true ability. One example

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is China's College Entrance Exam (CEE), where total score in the one-shot exam is used as the sole criteria for colleges to admit students applying for them (Wu (2007)). Widely-used Serial Dictatorship mechanisms can achieve ex-post fairness, i.e., matching students with higher scores to better (or commonly preferred) colleges. However, such an outcome is imperfect on ex-ante fairness, which suggests matching students with higher abilities to better colleges. Some researchers suggest that Boston mechanism with preference submission before exam (i.e., a BOS-before mechanism) may help to achieve ex-ante fairness (Lien et al. (2016, 2017)). This solution, however, is only partially justified either in theory or in lab. One problem is that, when a student can submit a *complete* preference list ove all colleges, he has a minimum-guarantee by his *non-first* choices, so he may list a college much better than qualified or 'fair' school as his first choice, and expect to be the 'lucky' one in the exam, i.e., to be admitted by his or her first choice. The incentive distortion will damage the ex-ante fairness. Based on such a reasoning, a mechanism with constrained school choice, where students can submit some but not all colleges in his preference list (Haeringer and Klijn (2009)), may help to correct such a distortion and achieve more ex-ante fairness (Lien et al. (2017)).

In this paper, we experimentally study the role of constained school choice in achieving ex-ante fairness under different matching mechanisms. We implement a 2-by-2-by-3 experimental design. Following Lien et al. (2016, 2017), we first categorize matching mechanisms by two dimensions. One concerns matching algorithm and has been widely discussed in the literature, i.e., the Boston (BOS) and the Serial Dictatorship (SD) mechanism. The other dimension is the preference submission timing: before the exam is taken and after the score are published. The resulting four mechanisms are therefore called BOS-before, BOS-after, SD-before and SD-after mechanisms. For each of those four mechanisms, we consider three levels of school choice constraint, i.e., unconstrained, moderately constrained, or fully constrained. Different levels of constraints are imposed simply by varying the number of colleges students are allowed to submit. Note that under the full constrained school choice where students can submit only one college, BOS and SD mechanisms are equivalent.

We propose and test two hypotheses: first, BOS-before outperforms the other three mechanisms on ex-ante fairness under *unconstrained* school choice. This hypothesis just repeats the hypothesis in Lien et al. (2016). Second, BOS/SD-before mechanisms can outperform BOS/SD-after mechanisms under *constrained* school choice. By comparing experimental results regarding those two hypotheses, we will have a better understanding of the role of constrained school choice on achieving ex-ante fairness.

The intuition for the first hypotheses is the following: under BOS-before mechanism, students have to submit their preference before the exam is taken. Therefore, students have to rely on their true ability, which is arguably the best estimation of their realized college entrance exam score, to submit their preference order. Ex-ante fairness will likely to be achieved, if all students' incentives are aligned by listing their ex-ante fair college as their first choice.

However, if one student considers to deviate from the above-mentioned cooperative strategy, he might benefit. He can put one college better than his ex-ante fair college as his first choice, while put the ex-ante fair college as his second choice. As long as he exceeds a student with higher ability by *realized* scores, he will get that student's fair college, which is supposed to be a better one than his own fair college. Otherwise he can still get his fair college. On average, the deviation is beneficial, and the proposed strategy profile leading to ex-ante fairness is not an equilibrium. One way to recover ex-ante fairness is to reduce the number of colleges a student is allowed to submit, so that he will face fewer non-first-choice (or back-up) colleges. If he still chooses to 'overstate' his first choice but fails to be admitted by it, he must be admitted by a worse college than when he has unconstrained choice set. Constrained school choice then serves as a commit device for students to put their ex-ante fair colleges as first choice, leading to ex-ante fair matching outcomes. This comes our second hypothesis.

Our research is related to two branches of literatures. One branch is literature reconsidering Boston mechanism under some forms of uncertainties. In earlier literature, the Boston mechanism have been commonly regarded as inferior to Serial Dictatorship mechanism on efficiency and (ex-post) fairness, see Abdulkadiroğlu and Sönmez (2003) and Ergin and Sönmez (2006). Abdulkadiroğlu et al. (2011), however, argue that when students have homogenous ordinal (but different cardinal) preferences over colleges, and colleges have random priorities over students, BOS can outperform SD on efficiency. Featherstone and Niederle (2014) argue that when both students have private information on their preferences and schools have random priorities, Boston mechanism can implement a truth-telling Bayesian Nash equilibrium and achieve more efficiency than deferred acceptance (DA) mechanism. Lien et al. (2016, 2017) argue that Boston mechanism with preference submission before exam can achieve more ex-ante fairness than other mechanisms. Our paper is a direct extension of those researches. In particular, we adapt experimental design of Lien et al. (2016), but impose constrained school choice to reconsider ex-ante fairness for the same mechanisms as in their paper.

Another branch of literature is constrained school choice. Haeringer and Klijn (2009) first raise this issue and study welfare consequence of it on Boston, DA and top trading cycles (TTC) mechanisms. Boston mechanism is immune from such a quota restriction, in the sense that it can always implement stable matchings for any quota, while other mechanisms may not. Calsamiglia et al. (2010) conduct experimental study on this issue and conclude that constraints reduce efficiency and stability for all mechanisms. Furthermore, they found that BOS is not more stable than DA and TTC mechanism. The constrained school choice literature until now are lack of elements of uncertainties, as we do by introducing preference submission timing. It is therefore interesting to see how the combination of these two "imperfections" would have the potential of welfare improvement.

The inspiration of our research comes from China's college admission system, where a noisy signal of students' true ability, college entrance exam score, is almost the only determinant of college priority. China's system is also moving from a BOS-before towards a SD-after mechanisms (Chen and Kesten, 2017; Lien et al., 2016, 2017). Along the way, constrained school choice remains a prominent feature of this system. Although there are over 1, 000 colleges and vocational schools, students are allowed to fill in at most dozens of them in their preference list. All those features are reflected in our experimental design.

The rest of the paper is organized as follows: We introduce our experiment design, theoretical predictions and hypotheses in Section two. In Section three, we present our experimental results on both matching outcomes and subject behavior. Section four concludes.

2. Experimental Design

Following Lien et al. (2016) and Lien et al. (2017), we first divide our treatments by two dimensions. One is by the matching mechanism (or more precisely, algorithm), i.e., the Boston (BOS) or the Serial Dictatorship (SD) mechanism. The other dimension is the preference submission timing: before exam is taken or after. Under preference submission before exam, we assume each student only knows score distribution of all students, but not realized score of anyone. Under preference submission after exam, exam scores are published so that each student knows realized scores of all students. We then have four treatments (or mechanisms): BOS-before, BOS-after, SD-before and SD-after.

Under each of those four mechanisms, we then introduce constraints on students' choice over colleges. In particular, the number of colleges allowed to submit in one's preference list is gradually reduced: at first, they can list all the possible colleges. Then the number of colleges allowed to be listed is reduced. During the last round, they are allowed to submit a preference list containing only one college. Except for the restriction on the number of colleges, students have freedom to list any colleges in any order. In our experiments, we have totally 3 colleges for matching, so the fully constrained, partially constrained and unconstrained school choice correspond to submitting at most 1, 2 or 3 colleges in preference list. This completes our 2*2*3 experimental design.

We measure ex-ante fairness of matching outcomes by two methods. First, we consider whether a matching outcome is (fully) ex-ante fair, i.e., does not contain blocking pairs at all. Here a blocking pair is defined as a student-college pair in which both have incentives to alter their current matching and re-match with each other (Balinski and Sönmez (1999)). The other measurement is the number of blocking pairs. The lower this number, the fairer a treatment is. To focus on the issue of ex-ante fairness, following the experiment design of Lien et al. (2016), we assume students share exactly the same preferences over colleges, both ordinal and cardinal. However, students have different score distributions, and their realizations are also always different from each other.

2.1. Environment

In our experiment, a matching system contains three colleges A, B, C, and three students 1, 2, 3. Every college only has one vacant seat, and each student can only be admitted by one college. The students' score distribution and payoffs of being admitted by different college are listed in Table 1 and Table 2, which are common knowledge among all subjects.

We now describe all the procedures used to match students and colleges under experimentation. We first categorize them by matching mechanism (BOS or SD) and preference submssiont timing (being before or after exam). For each of them, we then describe how we vary the number of colleges allowed to submit.

Table 1: Student score distributions

Score type	High score	Normal score	Low score	Avg. score
Probability	1/3	1/3	1/3	
Student 1	95	90	85	90
Student 2	91	86	81	86
Student 3	87	82	77	82

Table 2: Payoffs for being admitted

Admission of college	A	B	C
Student 1's payoff	30	25	15
Student 2's payoff	30	25	15
Student 3's payoff	30	25	15

BOS-before mechanism

Step 1. All students simultaneously submit a preference list containing n colleges.

Step 2. Each student gets to know his and the other two students' realized score, which is randomly and independently drawn according to everyone's score distribution.

Step 3. Each college considers the first choice of all the students. Colleges will admit the student with the highest realized score among those who list it as their first choice.

If n=1, then the algorithm stops. Students not admitted in Step 3 are finally unadmitted.

Step 4. Colleges having vacancies then consider the second choice of all the students remaining unadmitted. They will admit the student with the highest realized score among those who are unadmitted yet and list it as their second choice.

If n=2, then the algorithm stops. Students not admitted in Step 3-4 are finally unadmitted.

Step 5. Colleges having vacancies then consider the third choice of all the unadmitted students. They will admit the student with the highest realized score among those who are unadmitted yet and list it as their third choice.

The algorithm stops anyway.

When n=3, the mechanism is unconstrained BOS-before mechanism;

when n=2, it is partially constrained BOS-before mechanism; when n=1, it is fully constrained. Constraints on other three mechanisms are similarly defined.

BOS-after mechanism

Step 1. Each subject gets to know his and the other two students' realized score, which is randomly and independently drawn according to one's score distribution.

Step 2. All subjects simultaneously submit a preference list containing n colleges.

All the rest are the same as listed in BOS-before.

SD-before mechanism

Step 1 and Step 2 are the same as BOS-before.

Step 3. Student with the highest realized score among three will be admitted by his highest ranked college in his preference list.

Step 4. Student with the second highest realized score among three will be admitted by his highest ranked college having vacancies.

Step 5. Student with the third highest realized score among three will be admitted by his highest ranked college having vacancies.

Students not admitted in Step 4-5 are finally unadmitted.

SD-after mechanism

Step 1 and Step 2 are the same as BOS-after.

Step 3 to Step 5 are the same as SD-before.

2.2. Equilibrium

We now derive the (pure-strategy) Nash equilibrium for each treatment/mechanism decribed above.

2.2.1. Unconstrained mechanisms

Under unconstrained mechanisms, the equilibrium strategy and outcome has been figured out by Lien et al. (2016). We will only replicate their theoretical results here.

Conclusion 1.1. *Under unconstrained BOS-before mechanism: (1) the Nash equilibrium strategy profile for three students is of the form [(A,*,*), (B,*,*), (B,*,*)], where * denotes any college not listed yet. (2) The equilibrium outcome is [(1,A), (2,B), (3,C)] or [(1,A), (2,C), (3,B)], depending on whether student 2 gets a higher realized score than that of student 3.*

Conclusion 1.2. *Under unconstrained BOS-after mechanism: (1) The Nash Equilibrium is as follows: The student with the highest realized score* *submits (A,*,*), the student with the second highest realized score submits (B,*,*), the last one submits any list. (2) The equilibrium outcome is college A matching with the student with the highest score, college B matching with the student with the second highest score, college C matching with the last one.*

Conclusion 1.3. *Under unconstrained SD-before mechanism: (1) The Nash equilibrium is that all students play truth-telling strategy, i.e., (A,B,C). (2) Matching outcome is the same as BOS-after mechanism.*

Conclusion 1.4. *Under unconstrained SD-after mechanism: (1) There are two Nash equilibria. One equilibrium is as follows: The student with the highest realized score submits (A,*,*), the student with the second highest realized score submits (B,*,*), the last one submits any list. The other equilibrium is that all students play truth-telling strategy, i.e., (A,B,C). (2) The equilibrium outcome is unique and the same as BOS-after mechanism.*

Under unconstrained BOS-after, SD-before/after mechanism, *ex-post* fairness is achieved. That is, students with higher scores alway match to better colleges.

2.2.2. Partially constrained mechanisms

When students are allowed to submit a preference ordering list containing two colleges, equilibrium will change significantly under BOS/SD-before mechanisms. We summarize our findings for these two mechanisms in Proposition 1 and Proposition 2, with proofs provided in Appendix A.

Proposition 1. *Under partially constrained BOS-before mechanism: (1) The Nash equilbrium strategy profile for three students is of the form [(A,*), (B,C), (B,C)], where * denotes any college not listed yet. (2)The equilibrium outcome is the same as under unconstrained BOS-before mechanism.*

Proposition 2. *Under partially constrained SD-before mechanism: (1) The Nash equilibrium is of the form [(A,*), (B,C), (B,C)], where * denotes any college not listed yet. (2) The equilibrium outcome is the same as under unconstrained BOS-before mechanism.*

As for BOS-after and SD-after mechanisms, the equilibrium outcomes are the same as corresponding unconstrained mechanisms. The equilibrium strategy is somehow different due to the constraints. The formal description are as follows:

Conclusion 2.1. *Under partially constrained BOS-after mechanism: (1) The Nash equilibrium is that the student with the highest ex-post score submits (A,*), the student with the second highest ex-post score submits (B,*), the* *last one submits any list containing college C. (2) The equilibrium outcome is the same as uncostrained BOS-after mechanism.*

Conclusion 2.2. *Under partially constrained SD-after mechanism: (1) The Nash equilibrium is that the student with the highest ex-post score submits (A,*), the student with the second highest ex-post score submits (B,*) or (A, B), the last one submits any list containing college C. (2) The equilibrium outcome is the same as uncostrained BOS-after mechanism.*

2.2.3. Fully constrained mechanisms

When there is only one college allowed to be listed, BOS mechanism becomes the same as SD mechanism. We summarize our results for BOS/SD-before in Proposition 3, and the proof is provided in Appendix A.

Proposition 3. *Under fully constrained BOS/SD-before mechanism: (1)The Nash equilibrium strategy profile for three students is of the form [(A), (B), (C)] or [(B), (A), (C)]. (2) The equilibrium outcome is [(1,A), (2,B), (3,C)] or [(1,B), (2,A), (3,C)], respectively.*

It can be verified that a sufficient condition for BOS/SD-before to implment ex-ante fairness, stated in Proposotion 3.4 in Lien et al. (2017), is satisfied in our experimental set-up. Proposition 3, together with Conclusion 1.1 and 1.3, is consistent with Corollary 3.2 in Lien et al. (2017), which claims that fully constrained BOS/SD-before mechanisms are more likely to implment ex-ante fairness than unconstrained ones. However, it is true for only one of the two equlibiria stated in Proposition 3. Proposition 3 also claims the existence of multiple equlibria, which is not discussed in Lien et al. (2017). Specifically, in our experiment design, given the score distributions as specified, when $3u(B) \geqslant u(A) \geqslant 1.5u(B)$, and $3u(C) \geqslant u(B) \geqslant 1.5u(C)$, where $u(i)>0$ denotes payoffs admitted by college i, and non-admission payoffs are normalized to zero, there will be only one Nash Equilibrium [(1,A), (2,B), (3,C)]. Our set-up does not satisfy those inequalities, so there are multiple equilibria.

The eqilibrium outcome under BOS/SD-after mechanism is sitll not affected by imposed constraints:

Concluson 3.1. *Under fully constrained BOS/SD-after mechanism: (1)The student with the highest ex-post score submits (A), the student with the second highest ex-post score submits (B), the last one submits (C). (2) The equilibrium outcome is the same as uncostrained BOS-after mechanism.*

2.3. Theoretical predictions and hypotheses

From our equlibrium analysis in Section 2.2, we can derive the proportion of ex-ante fairness and the number of blocking pairs under different treatments/mechanisms. See Table 3.

Table 3: Theoretical predictions

Unconstrained	BOS-before	BOS-after	SD-before	SD-after
Probability of achieving fairness	2/3	10/27	10/27	10/27
Number of blocking pairs	1/3	7/9	7/9	7/9
Partially constrained	BOS-before	BOS-after	SD-before	SD-after
Probability of achieving fairness	2/3	10/27	2/3	10/27
Number of blocking pairs	1/3	7/9	1/3	7/9
Fully constrained	BOS-before	BOS-after	SD-before	SD-after
Probability of achieving fairness	1 or 0	10/27	1 or 0	10/27
Number of blocking pairs	0 or 1	7/9	0 or 1	7/9

For any given constraint level, BOS-before almost always (at least weakly) outperforms other three mechanisms on ex-ante fairness. The only exception is in one of two equilbria under fully constrained case, where BOS/SD-before perform acutually worse. It should be interesting to see how these two equilibria show out in the lab. Note also that equilibrium outcomes under BOS-before and SD-before converge under paritally constrained case (see also Proposition 2), although the mechanisms are not equivalent.

According to our theoretical predictions, in particular Table 3, we provide two hypotheses to be tested in our lab experiment.

Hypothesis 1 *Under unconstrained school choice, BOS-before outperforms other three mechanisms on ex-ante fairness.*

Hypothesis 2 *Under constrained school choice, BOS/SD-before outperforms other two mechanisms on ex-ante fairness.*

Hypothesis 1 restates the core hypothesis of Lien et al. (2016). However, they do not find any strong evidence to support this hypothesis. Hypothesis 2 parallels to Hypothesis 1, with the focus moving from unconstrained to constrained school choice. In addition, it claims the convergence of SD-before and BOS-before mechanism. We will find out whether and how constrained school choice would change the relative performance of those four mechanisms, by testing these two hypotheses.

2.4. Experimental implementation

We run four different sessions corresponding to BOS-before, BOS-after, SD-before and SD-after mechanism. In each session, subjects will experience all the three constraint levels, i.e., the unconstrained (listing three colleges), partially constrained (listing two colleges) and fully constrained (listing one college) environment sequentially. For each constraint level, every subject plays all the student type (student 1, 2 and 3) in turn. Subjects are randomly grouped in each round. All the subjects know their own matched college (therefore payoff) after each round, but the matching outcomes of other subjects are untold. Each subject totally plays $3 * 3 = 9$ rounds of the matching game. On finishing the 9 rounds of the matching game, they join a risk attitude test (Tanaka et al., 2010) and a personal information survey.

The payment paid in Chinese Yuan (RMB) are determined by their total payoffs in all sessions. The average payoff to each subject was 100 RMB, with a minimum of 72 RMB and a maximum of 120 RMB, including participation fee of 20 RMB. All the sessions were conducted on April 22nd of 2018. The number of subjects in each session was around 30 (min 27 and max 33). All the sessions were conducted in Tsinghua University, School of Economics and Management's Experimental Economics Laboratory (ESPEL) and all the subjects played the matching game on computer terminals.

3. Results

In this section, we show our main experimental results. Our focus is to investigate how each mechanism achieves ex-ante fairness in their matching outcomes. We also investigate individual behavior to explain how those matching outcomes are reached.

3.1. Ex-ante fairness

Following our theoretical prediciton in Section 2.3, in particular Table 3, we consider two measures subsequently: first is how likely a mechanism can achieve ex-ante fairness, i.e., proportion of ex-ante fairness; second is the number of blocking pairs averaged within a mechanism.

3.1.1. Proportion of ex-ante fair matchings

Figure 1-3, as well as Panel A of Table 4, shows the proportion of (fully) ex-ante fair matching outcomes under different mechanisms and different constraints. Under unconstrained case, proportions of ex-ante fair matchings are not significantly different among all four mechanism (at 90% level, statistical tests reported in Panel B, Table 4). In particular, the observed ex-ante fairness proportion under BOS-before is far from theoretical prediction (0.455 vs. 2/3), while the gap is smaller for the other three mechanisms (0.333 to 0.424 vs. 10/27). Our findings are consistent with Lien et al. (2016).

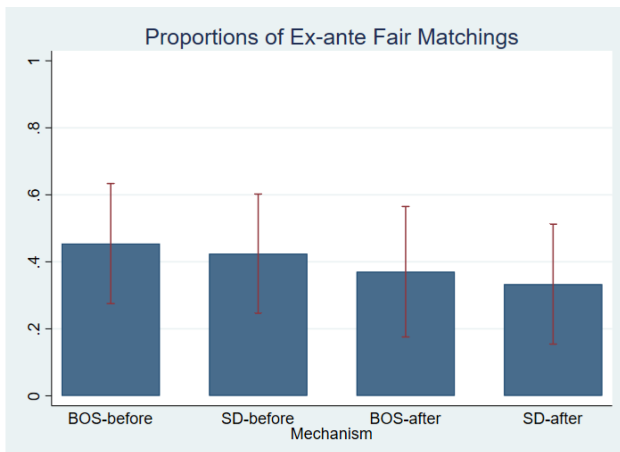

Figure 1: Proportion of ex-ante fairness under unconstrained mechanisms

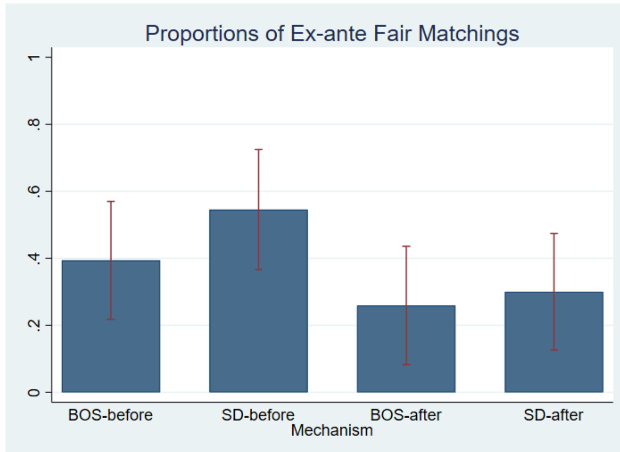

Figure 2: Proportion of ex-ante fairness under partially constrained mechanisms

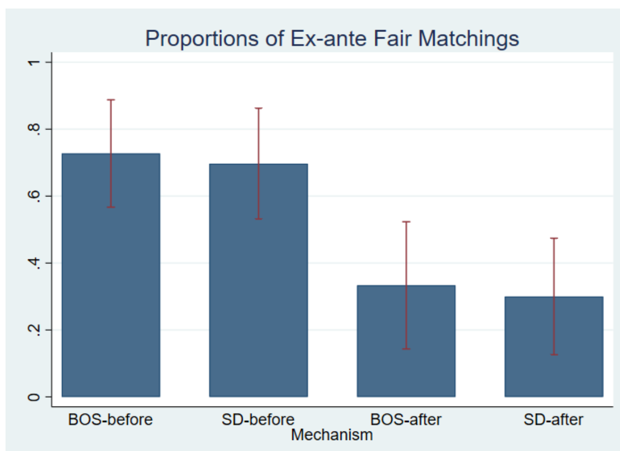

Figure 3: Proportion of ex-ante fairness under fully constrained mechanisms

Under partially constrained treatment, SD-before, but not BOS-before, stands out. In particular, differences between SD-before and BOS-after are significant at 95% level and differences between SD-before and SD-after are significant at 90% level, while other differences are not statistically significant (see Panel B, Table 4). Hyphotheis 2 is thus partially verified: only SD-before, but not BOS-before, dominates other two mechanisms on ex-ante fairness.

Table 4: Proportion of ex-ante fairness

	Panal A: Mean			
	BOS-before	BOS-after	SD-before	SD-after
Unconstrained	0.455	0.370	0.424	0.333
Partially constrained	0.394	0.259	0.545	0.3
Fully constrained	0.758	0.333	0.697	0.3

	Panal B: Wilcoxon Rank-Sum Test Result						
Difference	Bb-Ba	Bb-Sa	Bb-Sb	Sb-Sa	Sb-Ba	Sa-Ba	before-after
Unconstrained	0.084	0.121	0.030	0.091	0.054	−0.037	0.089
P-value	(0.514)	(0.329)	(0.806)	(0.462)	(0.674)	(0.772)	(0.319)
Partially constrained	0.135	0.094	−0.152	0.245	0.286	0.041	0.189
P-value	(0.275)	(0.438)	(0.221)	(0.051)	(0.027)	(0.735)	(0.032)
Fully constrained	0.424	0.457	0.061	0.497	0.364	−0.033	0.411
P-value	(0.001)	(0.000)	(0.583)	(0.002)	(0.005)	(0.789)	(0.000)

Under fully constrained treatment, proportions of ex-ante fair matchings under all the *before* mechanisms are higher than those under *after* mechanisms, significantly at 99% level. Within either *before* or *after* mechanisms, the differences are small and insignificant. Hyphothesis 2 are thus fully verified under fully constrained treatment.

Table 5 shows the same results from a different angle, i.e., by comparing matching outcomes among different constraint levels *within* each mechanism. Being fully constrained can increase ex-ante fairness significantly for *before* mechanisms, but not *after* mechanisms. Partial constraint (vs no constraint) does not affect ex-ante fainess within any mechanism. Hyphothesis 2 is still verified under fully constrained case, but not so under partially constrained case.

3.1.2. Number of blocking pairs

The results for the number of blocking pairs are shown in Figure 4-6, as well as Table 6. There are still no statistically significant differences among four mechanisms under unconstrained case. When student choice being partially constrained, SD-before plays statistically better than BOS-after, while other differences among four mechanisms are insignificant. However, when subjects are playing the matching game under full constrained treatment, although BOS-before still achieves a significantly better outcome than other mechanisms, SD-before turns out to be significantly worse than BOS-after. Yet *before* mechanisms still jointly perform better than *after* mechanisms (Table 6).

Table 5: Proportion of ex-ante fairness by constraint levels

	BOS-before	BOS-after	SD-before	SD-after	
Fully constrained	0.307	−0.035	0.267	−0.033	
(vs unconstrained)	(0.012)	(0.778)	(0.027)	(0.783)	
Partially constrained	−0.062	−0.109	0.119	−0.033	
(vs unconstrained)	(0.621)	(0.384)	(0.328)	(0.783)	
	before	after	BOS	SD	Overall
Fully constrained	0.286	−0.034	0.149	0.128	0.138
(vs unconstrained)	(0.001)	(0.692)	(0.102)	(0.153)	(0.030)
Partially constrained	0.030	−0.069	−0.086	0.049	−0.017
(vs unconstrained)	(0.728)	(0.422)	(0.348)	(0.588)	(0.794)

Table 7 compares matching outcomes across different constraint levels within given mechanisms. Imposing school choice constraints increases ex-ante fairness under BOS-before, decreases it under SD-before, both effects being insignificant. As a whole, constrained school choice does not show strong positive effects on decreasing the number of blocking pairs.

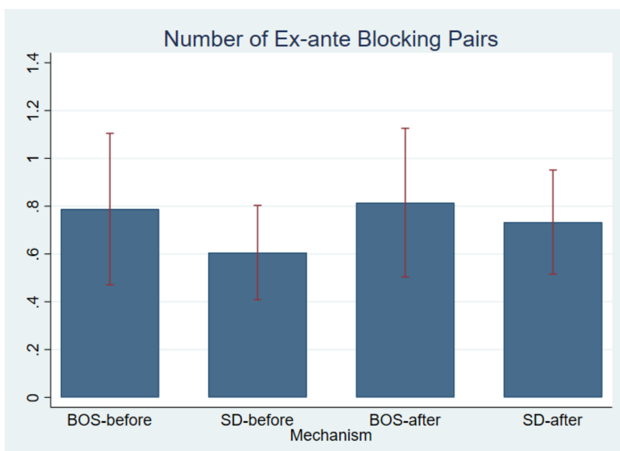

Figure 4: Average number of blocking pairs under unconstrained mechanisms

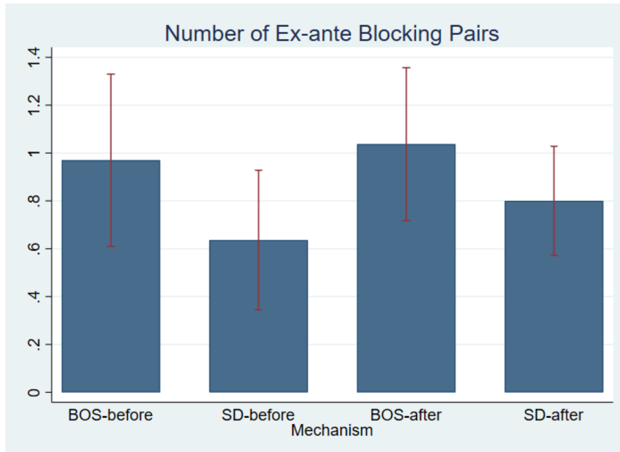

Figure 5: Average number of blocking pairs under partially constrained mechanisms

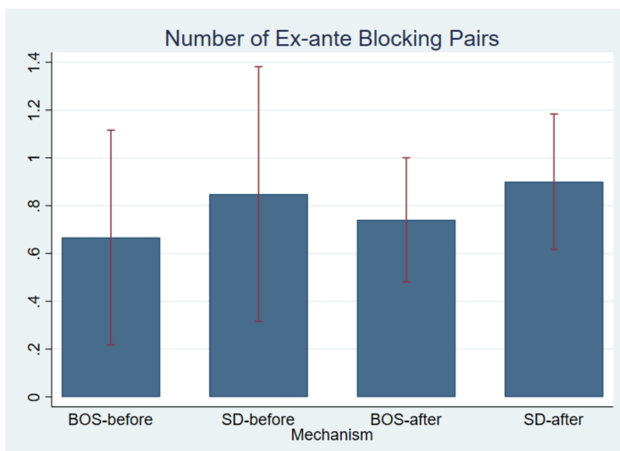

Figure 6: Average number of blocking pairs under fully constrained mechanisms

Our first measure of ex-ante fainess, i.e., proportion of ex-ante fair matchings, gives a strong support for Hypothesis 2 under fully constrained mechanism: imposing constraints on student choice over collges under BOS/SD-before mechanisms does increase ex-ante fairness and makes these two mechanisms dominate the other mechanisms. However, our second measure, the number of blocking pairs, does not support strongly Hypothesis 2. The difference between two measures worths further examinations. For a more

Table 6: Number of blocking pairs

	Panal A: Mean of number of blocking pairs			
	BOS-before	BOS-after	SD-before	SD-after
Unconstrained	0.788	0.815	0.606	0.733
Partially constrained	0.970	1.037	0.636	0.8
Fully constrained	0.636	0.741	0.848	0.9

	Panal B: Wilcoxon Rank-Sum Test Result						
Difference of blocking pairs	Bb-Ba	Bb-Sa	Bb-Sb	Sb-Sa	Sb-Ba	Sa-Ba	before-after
Unconstrained	−0.027	0.055	0.182	−0.127	−0.208	−0.081	−0.075
P-value	(0.730)	(0.809)	(0.646)	(0.393)	(0.369)	(0.872)	(0.414)
Partially constrained	−0.067	0.170	0.333	−0.162	−0.401	−0.237	−0.109
P-value	(0.528)	(0.840)	(0.173)	(0.172)	(0.040)	(0.276)	(0.180)
Fully constrained	−0.104	−0.263	−0.212	−0.061	0.108	0.160	−0.080
P-value	(0.029)	(0.011)	(0.567)	(0.044)	(0.092)	(0.403)	(0.003)

complete picture, we then look at the *distribution* of the number of blocking pairs.

Table 7: Number of blocking pairs by constraint levels

	BOS-before	BOS-after	SD-before	SD-after	
Fully constraint	−0.152	−0.074	0.242	0.167	
(vs unconstrained)	(0.077)	(0.807)	(0.226)	(0.453)	
Partially constraint	0.182	0.222	0.030	0.067	
(vs unconstrained)	(0.501)	(0.273)	(0.716)	(0.682)	
	before	after	BOS	SD	Overall
Fully constraint	0.045	0.053	−0.117	0.206	0.049
(vs unconstrained)	(0.037)	(0.730)	(0.155)	(0.772)	(0.221)
Partially constraint	0.106	0.140	0.200	0.048	0.122
(vs unconstrained)	(0.780)	(0.272)	(0.227)	(0.961)	(0.343)

Note: p-values are in parentheses, generated from Wilcoxon rank-sum tests.

3.1.3. Risk of mismatch

The histograms of the number of blocking pairs under various mechanisms and constraints are shown in Figure 7-9. In addition, we use two-sample Kolmogorov–Smirnov test to tell whether distributions are different from each other, shown in Table 8.

Under either unconstrained or partially constrained case, there are no significant differences on distributions of the number of blocking pairs between any two treatments. There is only one exception: under partially constrained treatment, SD-before has a different distribution from BOS-after, with a p-value of 0.088.

However, when considering fully constrained mechanisms, all the distributions between *before* and *after* mechanisms are significant different, with p-values of 0.020 for BOS-before vs BOS-after, 0.006 for BOS-before ve SD-after, 0.014 for SD-before vs SD-after and 0.039 for SD-before vs BOS-after. From Figure 9, we can see that distrbutions become more dispersed under *before* than under *after* mechanisms. In particular, the two *before* mechanisms have a higher possibility of reaching a highly unfair matching outcome with 3 or 4 blocking pairs.

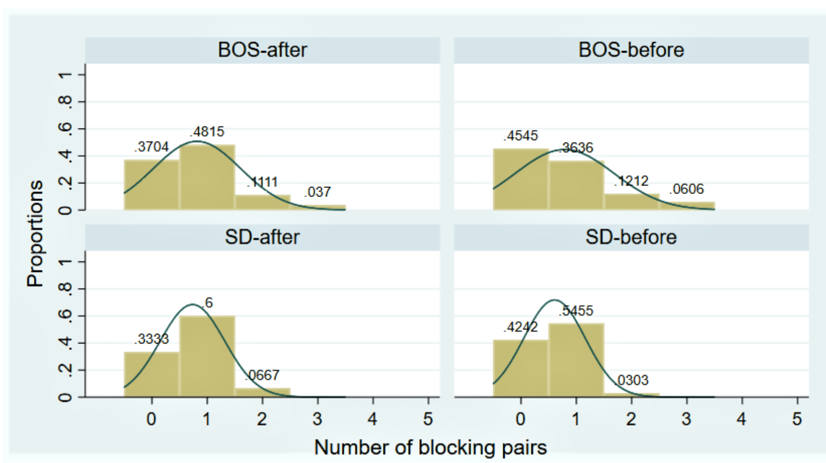

Figure 7: Distribution of number of blocking pairs under unconstrained mechanisms

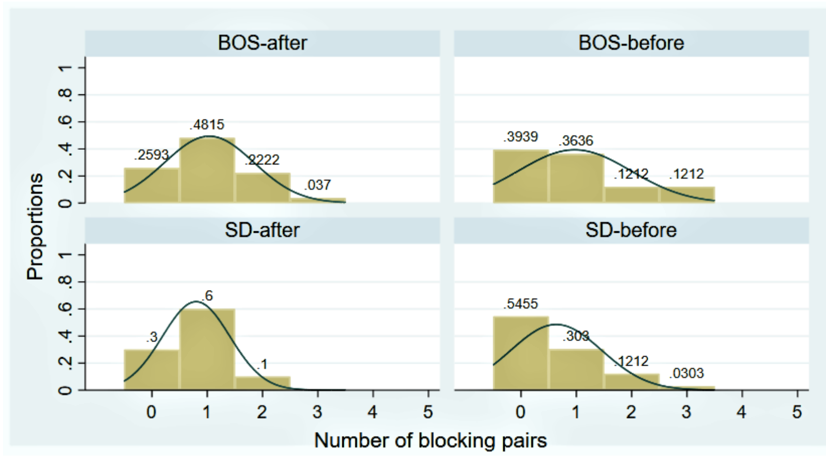

Figure 8: Distribution of number of blocking pairs under partially constrained mechanisms

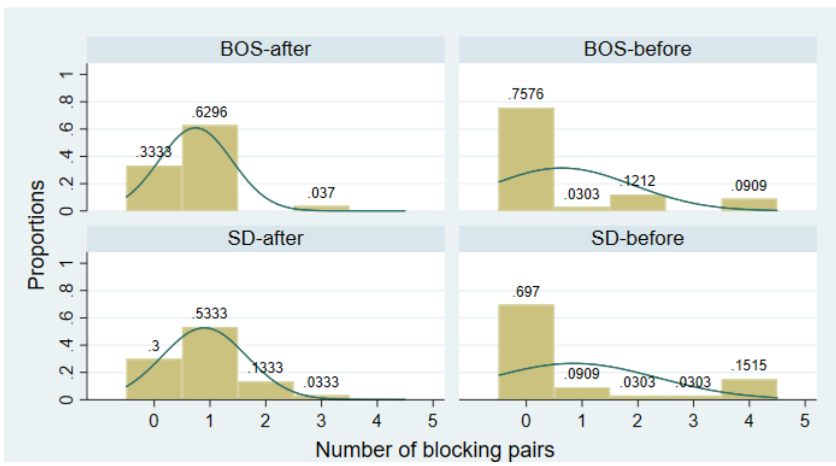

Figure 9: Distribution of number of blocking pairs under fully constrained mechanisms

Figure 10-12 show the cumulative distribution function (CDF) of the number of blocking pairs under different mechanisms for different constraint levels. Under unconstrained and partially constrained senarios (Figure 10 and 11), CDFs are not much different among four mechanisms. Under fully constrained senario, CDFs are quite different between *before* and *after* mechanisms. In particular, *before* mechanisms are more likely to achieve full ex-ante fainess, shown by larger CDF values when the number of blocking pairs being zero. Yet their advantage on achieving ex-ante fairness diminishes

Table 8: Difference in the distribution of number of blocking pairs

Unconstrained	Bb-Ba	Bb-Sa	Bb-Sb	Sb-Sa	Sb-Ba	Sa-Ba
The former contains smaller values	(0.810)	(0.630)	(0.970)	(0.771)	(0.662)	(0.828)
The latter contains smaller values	(0.967)	(0.659)	(0.469)	(1.000)	(1.000)	(0.962)
Combined results	(1.000)	(0.975)	(0.843)	(0.771)	(0.986)	(1.000)
Partially constrained	**Bb-Ba**	**Bb-Sa**	**Bb-Sb**	**Sb-Sa**	**Sb-Ba**	**Sa-Ba**
The former contains smaller values	(0.583)	(0.758)	(1.000)	(0.151)	(0.088)	(0.486)
The latter contains smaller values	(0.810)	(0.529)	(0.469)	(0.920)	(1.000)	(1.000)
Combined results	(0.950)	(0.907)	(0.843)	(0.300)	(0.175)	(0.864)
Fully constrained	**Bb-Ba**	**Bb-Sa**	**Bb-Sb**	**Sb-Sa**	**Sb-Ba**	**Sa-Ba**
The former contains smaller values	(0.005)	(0.001)	(0.761)	(0.007)	(0.020)	(1.000)
The latter contains smaller values	(0.402)	(0.771)	(1.000)	(0.486)	(0.402)	(0.620)
Combined results	(0.010)	(0.003)	(0.999)	(0.014)	(0.039)	(0.971)

Note: p-values are shown in parentheses, generated from two-sample Kolmogorov –Smirnov tests.

very quickly when the number of blocking pairs to be considered increases. In fact, the CDF values become smaller than other two mechanisms when the number of blocking pairs increase to only 1. Thus constrained school choice under BOS/SD-before mechanims implement ex-ante fairness more likely than other two mechanisms, with the cost of higher mismatch risk: they lead to highly unfair matchings with a higher probability.

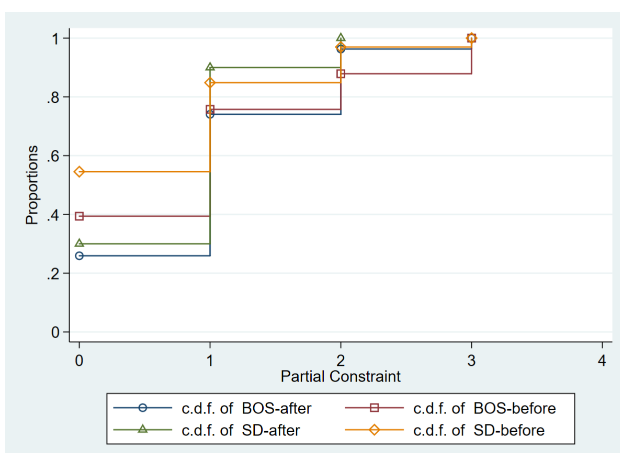

Figure 11: CDF of number of blocking pairs under partially constrained mechanisms

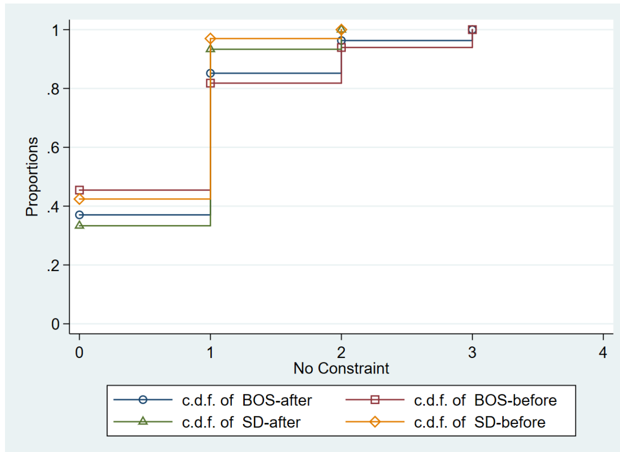

Figure 10: CDF of number of blocking pairs under unconstrained mechanisms

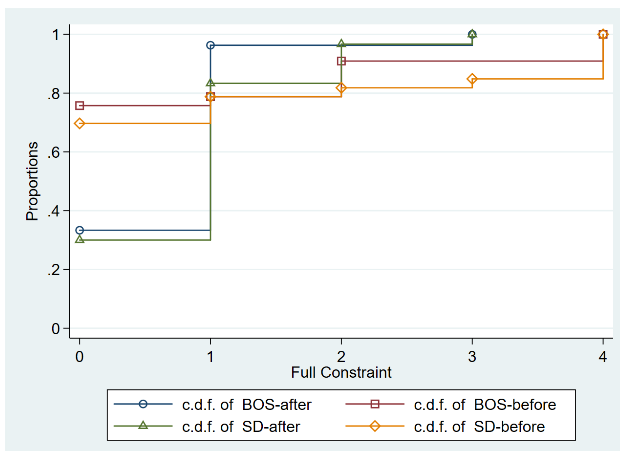

Figure 12: CDF of number of blocking pairs under fully constrained mechanisms

Table 9: Probability of non-admission under different mechanisms

Probability of non-admission	BOS-before	BOS-after	SD-before	SD-after
No constraint	0	0	0	0
Partial constraint	0.091	0.025	0.040	0.011
Full constraint	0.081	0	0.091	0

We further consider the probability of non-admission as an indicator of (un)fairness under different treatments. Non-admission must involve unfair matchings in our experimental set-up. It can also be a serious problem in reality. Non-admitted students must wait and work hard for another year for the CEE. Otherwise they will drop out from the system forever. Our results are shown in Table 9. Under unconstrained senario, there is no chance of being non-admitted for all mechanisms. Under partially constrained senario, the chance of non-admisson exists, with a level of less than 10 %, smaller under *after* mechanisms than under *before* mechanisms. The comparasion becomes sharper when we consider fully constrained senario. Here the chance of non-admission disapprears under *after* mechanisms, but sustains under *before* mechanisms. The outcomes confirm that *before* mechanisms are indeed riskier than *after* mechaisms for participants.

In short, the intuition behind this riskier outcome is as follows: On the one hand, there are several different Nash Equilibrium under our settings of fully constrained mechnisms and our subjects do not have chances to communicate with each other. Therefore, they may not to able to 'cooperate' although one of these Nash Equilibriums are more reasonable to some extent. On the other hand, *before* mechanisms with high level of constraints can cause non-admission problems, resulting in a relatively large number of blocking pairs, which also increase the possibility of mismatch.

In conclusion, although *fully constrained before* mechanisms are more likely to achieve (fully) ex-ante fair outcome, they are also more likely to cause highly unfair outcomes. They are more risky than other mechanisms.

3.2. Analyzing subject behavior

Our results reveal an obvious gap between theory and lab evidence over *equilibrium outcomes*, suggesting that subjectives may not play *equilibrium strategy* in the lab. Who do not play equilibrium strategy? How they affect matching outcomes? In this section, we discuss observed subject behaviors behind the matching outcomes.

We first examine how each student role plays an equilibrium strategy under various treatments. Among multiple equlibria, we focus on the one achieving the highest ex-ante fairness. In particular, under BOS/SD-before with full constraints, we focus on the equilibrium achieving (full) ex-ante fairness.

Table 10 shows the proportions of playing equilibrium strategy under each mechanism. In general, subjects play equilibrium strategy quite often. The proportion in most environments is over 80% or even 90%. However, there are quite a few cases the proportion is low.

Under unconstrained BOS-before mechanisms, there are only 55% of student 2 and 67% of student 3 playing equilibrium strategy. The results are consistent with Lien et al. (2016), and explain why BOS-before does not dominate others on ex-ante fairness.

The situation becomes even more prominent under partially constrained BOS-before mechanism, where only 46% of student 2 and 58% of student 3 plays the equilibrium strategy. Furthermore, there are 39% of student 2 submit (A,*) instead of (B, C) (not reported in the table). If student 2 submits a (A,*) while the other two students play the equilibrium strategy, i.e., (A, *) and (B, C), ex-ante fairness can not be achieved, because student 3 will be admitted by school B for sure. Therefore, a large-scale deviation by student 2 may be responsible for the gap between lab results and theoretical predictions. For student 3, equilibrium strategy is playing (B,C). However, we surprisingly notice that 1/3 of student 3 plays a (C,B), which is dominated by (B,C) (not reported in the table). Nevertheless, these deviations will not affect the matching outcomes much because studemt 3 will be admitted by college C almost for sure - less than 10% of student 1 and 2 list college C as first choice in the lab.

Students 2 also deviates by a large proportion under partially constrained SD-before mechanism. Only 42% of them play the equilibrium strategy (B,C), while all the rest submits a (A,*), and most of them choose (A,B) rather than (A,C) (not reported in the table). However, SD-before perform surpringly well under partially constrained case. It acutually outperforms all other mechanisms. Why is that? Under BOS-before partially constrained mechanism, the deviation of student 2 will generate a unfair outcome for sure. However, under SD-before, the probability of achiveing ex-ante fairness is still as high as 0.370 even for the non-equlibrium profile [(A,*)(A,B)(B,C)] . As a result, SD-before mechanism is not so sensitive to strategy deviations as for BOS-before, under paritial constrained mechanisms.

Under full constrained senarios, the proportion of playing equilibrium strategy is pretty high. The proportion is 1 or close to 1 under *after* mechanisms, and around 80-90% under *before* mechanisms. However, as we found in Section 3.1, under *before* mechanisms, a significant proportion of matchings turns out to be highly unfair. The observed discrepancy between behaviors and matching outcomes can only be explained by the fact that matching outcomes are highly sensentive to student behaviors: A small deviation may

Table 10: Proportions of playing equilibrium strategy

No constraint	BOS-before	BOS-after	SD-before	SD-after
Student 1	0.939	0.852	1	1
Student 2	0.545	0.889	0.909	1
Student 3	0.667	0.963	0.727	1
Partially constrained	**BOS-before**	**BOS-after**	**SD-before**	**SD-after**
Student 1	0.939	0.926	1	1
Student 2	0.455	1	0.424	0.967
Student 3	0.576	0.963	0.788	0.967
Fully constrained	**BOS-before**	**BOS-after**	**SD-before**	**SD-after**
Student 1	0.909	1	0.879	1
Student 2	0.879	1	0.788	1
Student 3	0.848	0.963	0.909	1

result in a highly unfair matching outcomes (e.g., non-admission), simply because there is no any back-up choice under such mechanisms.

Why students may deviate from the equilibrium strategy? In Table 11, we consider how the risk attitude of subjects affect their choice. Lien et al. (2016) show that under unconstrained mechanisms, risk attitudes do not affect student behaviors. Here we focus on constrained mechanisms. In particular, we consider three cases where students deviate most, i.e, student 2 and 3 under BOS-before, and student 2 under SD-before, all under partially constrained mechanisms. Their risk attitudes are measured by three parameters (α, λ, σ) based on Tanaka et al. (2010). In these three parameters,σ and λ measure one's value function and level of loss aversion, α indicates curvature of the gain and loss segment. As shown in the table, α affects the behavior of student 2 under *partially constrained before* mechanisms significantly. All other risk parameters are not significant in the regressions, and risk parameters are jointly insignificant in all regressions. We conclude that risk attitude only plays a very limited role in affecting student behavior.

An alternative reason for deviaton might be that subjects may not be able to figure out the equilibrium strategy. Under BOS mechanisms and constrained SD mechanisms, students do not have dominant strategy, e.g., truth-telling. When preference submission is set before exam, they have to take into account of score distributions to calculate expected payoffs for any strategy profile. The task is almost impossible to complete. This cognitive restriction argument can explain why only *before* mechanisms suffer from strategic deviations.

Table 11: Explantory power of risk parameters to playing equilibrium strategy under partial constraints

	Dependent variable: Whether playing equilibrium strategy		
	Student 2 under B-b	Student 2 under S-b	Student 3 under B-b
σ	−0.293	−0.578	0.180
	(0.524)	(0.131)	(0.662)
λ	0.058	−0.056	0.001
	(0.309)	(0.248)	(0.987)
α	1.062	0.623	0.488
	(0.060)	(0.098)	(0.312)
Joint sig. of risk parameters (Prob>chi2)	0.233	0.269	0.470
Num of obs	33	33	33
Pseudo R^2	0.094	0.087	0.056

Note: p-values are shown in parentheses, generated from probit regressions. Independent variables are constant term, α, λ, σ, collected in the survey. Coefficient has been converted to dF/dx, meaning it measures the discrete change of playing equilibrium behavior from 0 to 1.

4. Conclusion

Colleges want to select students with high abilities. But student ability is unobservable, and colleges can only observe a noisy signal for it (e.g. exam scores). A carefully-designed mechanism, by properly incorporating those signals, may induce students to self select by their abilities and achieve a socially desirable matching outcome. In this paper we combined two design features, i.e., preference submission timing and constrained school choice, to explore potential improvement on true ability revealing, which can help us to achive assortative matching between school qualities and student abilities.

We conduct experiments under three constraint levels (full, partial or zero), two mechanisms (BOS or SD) and two preference submission timings (before-exam or after-exam). Our findings can be summarized as one conclusion but with two cautions. The conclusion is that BOS and SD mechanism under preference submission before exam with full constraints can achieve ex-ante fairness more frequently than other mechanisms. The first caution is that *partial* constraints do not work as well for achieving ex-ante fairness. The second caution is that BOS/SD-before with full constraints can be risky for players, i.e., lead to a highly unfair matchings, albeit with a small probability. Here the typical trade-off under asymmetric information, i.e, trade-off between incentive and insurance, emerges: to discipline agents to behavior well (i.e., to reveal their true types), they have to be less insured.

Our experiment design is 'small-scale', in which there are only three students and schools in the matching system. How can those results extend to a large-scale matching system (e.g. college admission in China). First, in the large-scale system, competitions between students may not necessarily become more fierce. Although the number of students increase, those who have competition relations with each other (i.e., overlapped realized scores) may still fall into a small group. Even if the competitive group becomes larger, a specific college often has as many slots as to accomodate competitting students. Second, as the number of colleges increase, the number of equilibria may increase as the preference order becomes longer. However, an equilibrium in which each student lists his ex-ante fair school as first choice may still be the focal point, under some proper mechanisms. Although, in a large-scale matching, figuring out one's fair school can be sophisticated, such complexity may not be invincible. Note that in our small-scale matching in the lab, subjects only have 10 minutes to understand the experimental setup, including their score distributions, ordinal preferences and the matching procedures. In reality, students spend plenty of time preparing for college admission, gathering information on their true abilites and eligible colleges. Under well-designed mechanisms, the system may hopefully get close to exante fairness.

Almost every province in China has reformed its admission rules by switching from a BOS-before mechanism to a SD-after mechanism. SD-after can generate an ex-post fair matching outcome, yet the 'true' ability of students may be hidden under the noisy signal of exam scores. The issue seems more severe in recent years. As the CEE difficulty decreases[1](People's Daily Online (2013)), the CEE scores may reflect students' true ability as much as their fortunes. The current reform thus can be questioned. Our paper suggests that the 'old' system, i.e, BOS-before with a sufficiently high constraint level, may have its own advantage, although it is also imperfect.

We call on further study on designing a better system.

Appendix A. Proof of propositions

Proof of Proposition 1

With the score distribution presented in Table 1, the probability of different score-rank is shown in Table A.12.

Table A.12: Probability of different score rank

Ranking of students by ex-post score	Probability
(1,2,3)	10/27
(1,3,2)	7/27
(2,1,3)	7/27
(2,3,1)	1/27
(3,1,2)	1/27
(3,2,1)	1/27

Claim 1.1 *There must be at least one student listing school A as his first choice in a Nash Equilibrium.*

proof. Under Boston mechanism, if there is only one student listing school A as his first choice, he will be admitted by school A for sure. Therefore, if no one chooses A as his first choice, any student has incentives to change his first choice to A to get a 30, which is the largest possible payoff.

Claim 1.2 *No one lists school A as one's second choice.*

proof. By Claim 1.1, there must be at least one student listing A as his first choice, therefore, listing A at the second place will have 0 probability of being admitted by school A. Listing any other school as the second choice will be better than listing A.

Claim 1.3 *There is no equilibrium that two students submit (C,B).*

[1]e.g., the minimum guaranteed score of being admission by first tier of colleges in Beijing increased significantly in 2013. In 2008-2012, that score for science was between 477-502 (out of 750) and it lied in 532-550 in 2013-2018. On the other hand, the score for arts lied in 495-532 in 2008-2012 and increased to 549-583 in 2013-2018. Also see http://edu.people.com.cn/n/2013/0623/c1053-21939237.html for news from People's Daily Online(in Chinese) about the increase of admission score in 2013.

proof By Claim 1.1, there must be at least one student listing A as his first choice. If the other two students submit (C,B), then any one of them has incentives to change to (B,C) to get 25 for sure.

Claim 1.4 *(C,B) is dominated by (B,C).*

Proof By Claim 1.2, there are only four possible strategy left, that is (A,B) (A,C) (B,C) (C,B). By Claim 1.1 and 1.3, submitting (C,B) will get a 15 payoff for sure. If there is no other students submitting (B,C), then he can change to (B,C) to get a 25 instead. If there is a student submitting (B,C), changing his choice to (B,C) will get a payoff of $\theta * 25 + (1 - \theta) * 15$, where $0 < \theta < 1$. Therefore, submitting (B,C) always dominates submitting (C,B).

Claim 1.5 *There is no equilibrium that all students list A as first choice.*

proof By Claim 1.1-1.4, there are only three possible strategy left, that is (A,B) (A,C) (B,C). If all students list A as first choice, there are two cases in total.

Case 1. All students submit (A,B). Under this case, we can easily calculate the payoff of student 3, which is 2/27 * 30 + 8/27 * 25 = 260/27 < 15. Therefore, student 3 has incentives to choose B or C as his first choice.

Case 2. Not all students submit (A,B), which means there is at least one student submitting (A,C). Considering the student who choose (A,C), even if he is student 1, his payoff can not be larger than 17/27 * 30 + 10/27 * 15 = 660/27 <25. Therefore, those submitting (A,C) have incentives to submit (B,C) instead.

By Claim 1.5, with only three possible strategy (A,B) (A,C) (B,C) left, there are now 5 strategy combination left.

Combination 1 (A,B) (A,B) (B,C)

Combination 2 (A,B) (A,C) (B,C)

Combination 3 (A,C) (A,C) (B,C)

Combination 4 (A,B) (B,C) (B,C)

Combination 5 (A,C) (B,C) (B,C)

Claim 1.6 *No one chooses (A,B) when only one student submitting (B,C).*

proof. When only one student submits (B,C), implying the other two list A as their first choice, he will get 25 for sure. If one student chooses (A,B), his second choice on the list is wasted and will certainly decrease his payoff, since he has a positive probability of being not admitted by school A. Submitting (A,C) instead will increase his expected payoff for sure.

Claim 1.7 *No one chooses (B,C) when the other two students submitting (A,C).*

proof. When only one student submits (B,C) and the other two submit (A,C), the one who submits (B,C) will get 25 for sure. However, he will still be the only one who lists B in the preference ordering list if he submits (A,B) instead. Changing to (A,B), his expected payoff will be $\alpha * 30 + (1 - \alpha) * 25$, where $0 < \alpha < 1$. Therefore, (A,C) dominates (B,C) when the other two submit (A,C)

Claim 1.8 *Under combination 4 and 5, in any equilibrium, student 1 submit (A,*).*

proof. If (A,*) is submitted by student 2, meaning student 1 and student 3 are submitting (B,C) currently. Student 3 has incentives to submit (A,C) instead. His expected payoff will be 1/3 * 30 + 2/3 * 15 = 20 > 1/9 * 25 + 8/9 * 15 = 145/9. If (A,*) is submitted by student 3, meaning student 1 and student 2 are submitting (B,C) currently. Student 1 has incentives to submit (A,C) instead. His expected payoff will be 8/9 * 30 + 1/9 * 15 = 85/3 > 2/3 * 25 + 1/3 * 15 = 65/3.

By Claim 1.6 and Claim 1.7, combination 1, 2, 3 have been dropped. By Claim 1.8 combination 4 and 5 must have student 1 submitting (A,*). Therefore, the only possible equilibrium has the form [(A,*), (B,C), (B,C)]. It is straight forward to verify that such a strategy profile indeed forms a Nash Equilibrium.

With the result listed above, student 1 will be admitted by school A for sure, and student 2 and 3 will be admitted by school B or C, depending on who gets a higher score.

Proof of Proposition 2

Claim 2.1 *In any equilibrium, the better the school, the higher its rank on any student's preference ordering list.*

proof. Under SD mechanism, a student will be admitted by his highest ranked school remaining unmatched. Therefore, for any two schools a student want to include in his preference list, it is always weakly better to rank the preferred one higher. And since every student has a positive probability to be admitted by his first choice, it is then strictly better to rank the preferred one higher.

Claim 2.1 means that no one will lie (e.g. submit (B,A)).

Claim 2.2 *In any equilibrium, the preference ordering list of three students are not all the same.*

proof. If all of them submit the same preference, the expected payoff of student 3 is no more than 2/27 * 30 + 8/27 * 25 = 260/27 < 15. But he can always get at least 15: If the other two do not include school A in their list, he can get 30 by listing (A,*); or if the other two do not include B in their list, he can get 25 by listing (B,C); otherwise he can get 15 by listing (A,C).

By Claim 2.1, there are only 3 possible strategy for a student, that is (A,B), (A,C), or (B,C).

Claim 2.3 *In any equilibrium, student 1 will not submit (B,C).*

proof. Suppose student 1 submits (A,B). When he ranks first (with prob.=17/27), he gets 30. When he ranks second (with prob.=8/27), he at least gets 25. So his payoff by submitting (A,B) is at least 17/30 * 30 + 8/27 * 25 =710/27 > 25. But his payoff by submitting (B,C) can not be larger than 25. So he will never submit (B,C).

Claim 2.4 *If student 1, 2 submit (A,*) but not the same list, student 3 will submit (B,C), instead of any form of (A,*).*

proof. We only need to prove that submitting (B,C) is alway better than submit (A,*) for student 3. After some calculations, one can see that among submitting (A,B) and (A,C), submitting (A,B) is better for student 3, no matter who submits (A, B) (or (A, C)) among student 1 and 2. When student 3 submits (A,*), he will get A when he ranks first, and his second choice ('*') when he ranks second. The advantage of submitting (A,B) instread of (A,C) is largely due to the 'secong choice' advantage.

By comparing submitting (A,B) and (B,C), we can find that submitting (B,C) is alway better. By submitting (B,C), student 3 will always be admitted. However, if he submits (A,B), he will not be admitted with some probability. The advantage of submitting (B,C) instead of (A,B) is largely due to this 'admission' advantage.

Claim 2.5 *The strategy profile [(A,C), (A,C), (A,B)] is not an equilibrium.*

proof. It can be easily verified that under this strategy profile, student 1 has the incentive to deviate to (A,B).

Claim 2.6 *The strategy profile [(A,B), (A,B), (A,C)] is not an equilibrium.*

proof. It can be easily verified that under this strategy profile, student 3 has the incentive to deviate to (B,C).

By Claim 2.4-2.6, the strategy profile of any form of [(A,*), (A,*), (A,*)] is not an equilibrium. Then at least one of student 2 and 3 will submit (B,C).

Claim 2.7 *If student 1 submits (A,*), and student 2 submits (B,C), then student 3 will submit (B,C).*

proof. This can be easily verified.

Claim 2.8 *If student 1 submits (A,*), and student 3 submits (B,C), then student 2 will submit (B,C).*

proof. This can be easily verified.

Therefore, the strategy profile [(A,*), (B,C), (B,C)] is the only form of pure-strategy Nash Equilibrium.

Proof of Proposition 3

Claim 3.1 *In any equilibrium, no students list the same school.*

proof. If two or more students submit the same list, the one with the lowest ex-ante average score among them only has a probability of no more than 1/3 to be admitted by that school. Therefore, his expected payoff can not be more than $1/3 * 30 = 10 < 15$. Meanwhile, there must be at least one school which no students list. He has incentives to deviate and submit that unlisted school to get 15, 25 or 30, depending on which school is not listed by anyone.

Claim 3.2 *In any equilibrium, student 3 does not list school A.*

proof. Suppose not. By Claim 3.1, every student will be admitted by its listed school. If student 3 list school A, then he will get a 30 payoff for sure. Considering the one admitted by school C, he is now getting 15 and he has incentives to challenge student 3. If he is student 1, his expected payoff will become $30 * 8/9 = 80/3 > 15$. If he is student 2, his expected payoff will become $30 * 2/3 = 20 > 15$.

By Claim 3.1 and Claim 3.2, it then can be easily verified that two Nash equilibria exist: [(A), (B), (C)] or [(B), (A), (C)].

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